MATERIALS

# Reliable Ceramic Structural Composites Designed with a Threshold Strength

## Technical Report # 2

## Optimal Threshold Strength of Laminar Ceramics

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published in: Z. Metallkd. 90[12] 1999

Period: 5/1/99 through 4/30/00

Office of Naval Research

Grant No. N00014-99-1-0638

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# Optimal Threshold Strength of Laminar Ceramics

*Dedicated to Prof. Frederick F. Lange on the occasion of his 60th birthday*

Laminar ceramics in which alternating layers of material are bonded together exhibit a threshold strength if one set of layers has a compressive residual stress. The threshold strength is substantial if the mechanical properties of the component ceramics are chosen wisely. Optimization of the threshold strength for a system with homogeneous elastic properties in terms of the layer thicknesses, fracture toughness and residual stress is presented. The best result is shown to be associated with the toughest material and the highest residual stress, but the choice of a laminar system exhibiting such features is limited to available ceramics. For each material system, the threshold strength is further optimized by making the layers as thin as possible. The thinness achievable will be limited by the technological processes used to make the laminar ceramic and by material stability. Given a laminar ceramic system with a specific compressive layer thickness, the threshold strength is optimized by selecting a ratio of tensile to compressive layer thickness. If the system mechanical properties are favorable, the optimized threshold strength can be comparable in magnitude or much larger than the compressive residual stress.

## 1 Introduction

Fred Lange has been involved in many exciting and important developments in the field of ceramics and the senior author of this paper is privileged to have been associated with a few of his efforts in this area. Fred's infectious enthusiasm for research on mechanical properties and processing of ceramic materials is an attractive element of any interaction with him. His deep and wide-ranging knowledge of the field is a treasured resource for those of us who have the honor of working with him. Fred's skill of explaining sophisticated phenomena clearly in terms of simple materials science, physics and chemistry makes the labor of learning from him an easy task, even for those of us trained only in solid mechanics. It is with admiration, friendship and gratitude that we dedicate this paper to honor Fred Lange's 60th anniversary.

Recently, Fred has discovered that laminar ceramics can be designed to provide a threshold strength below which failure due to monotonic loading is impossible [1]. This development is of far reaching importance since engineers will now be able to design ceramic components to be completely reliable. This contrasts to the current state of the art where ceramic components exhibit a distribution of strengths and some must be expected to fail at quite low loads. Absent a proof test, this presents a situation in which ceramics cannot be used in critical components due to lack of reliability. However, if a threshold strength is a robust feature of a ceramic component, a design for it can be developed that assures absolute reliability and safety. This breakthrough in ceramic engineering will have far reaching consequences in the application of these brittle materials.

The laminar ceramic is designed to have alternating layers of material as shown in Fig. 1, with compressive layers sandwiched between tensile layers. The tensile loading direction is parallel to the layers so that the strength controlling cracks are transverse to the slabs as depicted in Fig. 1. Since the layered ceramic is made from slabs fused together, any flaw present at the outset is assumed to be confined entirely within one layer. As in any brittle composite material, this feature by itself has beneficial effects on the performance of the material since the strength controlling cracks are limited in size by the scale of the elements in the system, in this case the layers. If a flaw lies in the compressive layer, there is a lower bound to the applied tensile stress necessary to propagate the crack. This phenomenon is completely analogous to the behavior of surface cracks in ceramics with compressive layers on their perimeter [2 to 7]. However, the catastrophic growth of cracks which are present initially in the tensile layers is also absent below a threshold strength [1] and so all cracks in the layered system are limited in their tendency to propagate under applied stress.

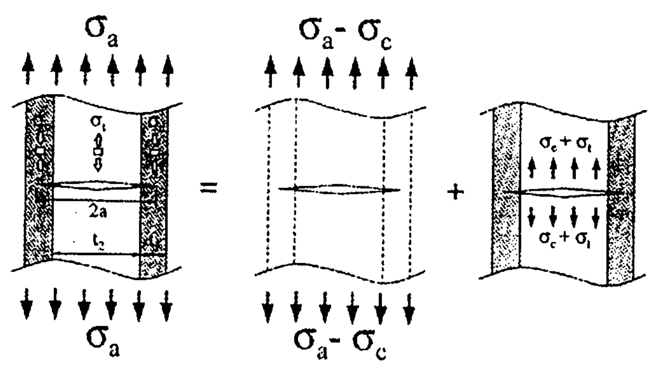


Fig. 1. A laminar ceramic loaded parallel to the interfaces between the layers and having a through crack in a tensile layer which partially penetrates the compressive layer. The scheme for calculating the stress intensity factor is suggested by the loads decomposed into two contributions.

It will be shown that the threshold stress for propagation of the cracks in the tensile layers is lower than that for propagation of cracks in the compressive layers. Since propagation of cracks in the tensile layers can occur at lower applied stresses than those which will cause growth of cracks in the compressive layers, the flaws in the tensile slabs control the true threshold strength of the system. Rao et al. [1] have shown that this threshold strength depends on the toughness of the material in the compressive layers, the thicknesses of the layers and the residual stress in the compressive layer. It is of interest to determine how to choose these parameters to obtain the largest possible threshold strength for the laminar ceramic. Such an effort is the subject of this paper.

## 2 Threshold Strength

A crack in the tensile layer will grow towards the compressive layer when the stress intensity factor $K$ due to the combination of the applied and the tensile residual stress reaches the fracture toughness of the material in the tensile layers. Strength limiting flaws in the tensile slab are those that have a length which is a substantial fraction of the layer thickness. These cracks will grow at relatively low applied stress, which we assume to be below the threshold strength of the laminar system. When the tip of such a crack enters the compressive layer, the growth into the compressive material will stop before it grows all the way across. Instead, at more or less the same applied stress, the flaw will tunnel down the length of the tensile layer to form a through crack as depicted in Fig. 1 [8].

Further growth of the crack as depicted in Fig. 1 will take place when the stress intensity factor $K$ equals the toughness $K_c$ of the material in the compressive layer. Rao et al. [1] have shown that the stress intensity factor for a laminar system with isotropic components having the same elastic properties is

$$K = \sigma_a\sqrt{\pi a} + \sigma_c\sqrt{\pi a}\left[\left(1+\frac{t_1}{t_2}\right)\frac{2}{\pi}\sin^{-1}\left(\frac{t_2}{2a}\right)-1\right] \quad (1)$$

where $\sigma_a$ is the applied stress, $2a$ is the crack length, $\sigma_c$ is the magnitude of the compressive residual stress in the layer of thickness $t_1$ and $t_2$ is the thickness of the layer with tensile residual stress. (It should be noted that this result is strictly valid only if the different layers have the same elastic properties.) If the stress intensity factor falls as the crack extends in the compressive layer, the flaw will grow stably as the applied stress is increased until it reaches the next tensile layer. By differentiating Eq. (1) with respect to $a$, setting $K$ equal to $K_c$ and considering a crack which spans two compressive layers and the tensile layer in-between, we find that stable growth across the entire compressive layer occurs as long as

$$\sigma_c \geq \frac{\pi K_c}{2\sqrt{\pi t_1\left(1+\frac{t_2}{2t_1}\right)\left(1+\frac{t_2}{t_1}\right)}} \quad (2)$$

If this condition is met, Rao et al. [1] have shown that the stress $\sigma_{th}$ to drive the crack all the way through the compressive layer is

$$\sigma_{th} = \frac{K_c}{\sqrt{\pi\frac{t_2}{2}\left(1+\frac{2t_1}{t_2}\right)}} + \sigma_c\left[1-\left(1+\frac{t_1}{t_2}\right)\frac{2}{\pi}\sin^{-1}\left(\frac{1}{1+\frac{2t_1}{t_2}}\right)\right] \quad (3)$$

Since the flaw has reached this length in a stable manner and will propagate unstably beyond here, Eq. (3) is the estimate of the threshold strength controlled by cracks initially in the tensile layer, as long as the condition in Eq. (2) is met. This situation does not always prevail and a threshold strength for cases violating Eq. (2) will be considered below.

Consider now flaws initially in the compressive layers. Those with a diameter close to the layer thickness will propagate first. Tunneling down the length of the compressive layer and propagation into the tensile layers in an unstable manner will occur more or less at the same applied stress $\sigma_{cth}$, which can be estimated as

$$\sigma_{cth} = \frac{K_c}{\sqrt{\pi\frac{t_1}{2}}} + \sigma_c \quad (4)$$

Comparison of Eq. (3) and Eq. (4) shows that $\sigma_{cth}$ exceeds $\sigma_{th}$ so the latter is the true threshold strength of the laminar ceramic, given that Eq. (2) is satisfied.

## 3 Optimal Threshold Strength

The optimization of the threshold strength requires the maximization of $\sigma_{th}$ with respect to the fracture toughness, the layer thicknesses and the residual stress in the compressive layers. However, the residual stress and the layer thicknesses are not independent, so we must make use of the following result [1] for a laminar system in which the different layers have the same elastic properties:

$$\sigma_c = \frac{-E'\varepsilon}{1+\frac{t_1}{t_2}} \quad (5)$$

where $E' = E/(1-\nu)$ is the biaxial elastic modulus, $E$ is Young's modulus, $\nu$ is Poisson's ratio and $\varepsilon$ is the differential strain between the compressive and the tensile layers. The source of the differential strain can be thermal expansion mismatch or a transformation strain, so no single formula can be given to represent it. However, in the case of thermal expansion mismatch,

$$\varepsilon = (\alpha_2 - \alpha_1)\Delta T \quad (6)$$

where $\alpha_1$ is the coefficient of thermal expansion of the material in the compressive layer, $\alpha_2$ is the coefficient of thermal expansion of the material in the tensile layer and $\Delta T$ is the current temperature minus the temperature at which the layers are free of residual stress. Note that the thermal expansion coefficients are assumed to be independent of temperature in this example and $\varepsilon$ is a negative number.

Inspection of Eq. (3) together with Eq. (5) reveals that the threshold strength is maximized by selecting the materials that give the highest $K_c$, $E'$ and $\varepsilon$. Selection is, of course, limited to available combinations of materials and the best choice to achieve the highest threshold strength can be determined only by identifying the parameters of the materials and evaluating Eq. (3). A useful methodology for this would be to construct maps in the fashion of Ashby [9] and identify the most promising combinations of materials by their position on the map. This activity will be left to a future effort.

## Unconstrained Optimization of the Layer Thickness Ratio

After a good combination of materials has been selected and thus the values of $K_c$, $E'$ and $\varepsilon$ have been identified, the next step is to choose the layer thicknesses. It makes sense to use layers that are as thin as possible, since this will promote the first term on the right-hand side of Eq. (3). However, it is convenient to consider first a choice of $t_1$ that is purely arbitrary and then optimize the threshold strength in terms of the ratio $t_2/t_1$. This situation would arise when the compressive layer material is available only in sheets of a specific thickness so that this dimension cannot be adjusted at will. If the tensile layer material can be made to any thickness, then optimization of the threshold strength can be achieved by adjusting this dimension. The converse situation may also arise where the tensile layers are available only in sheets of a specific thickness but the compressive layer thickness can be adjusted. The optimization of the threshold strength in this situation is also an interesting problem, but for brevity it will not be considered here.

Having identified for a laminar ceramic system the values of $K_c$, $E'$, $\varepsilon$ and an arbitrary choice for $t_1$, we conclude that only the ratio $t_2/t_1$ remains as an independent unknown variable for optimizing the threshold strength. It follows that Eq. (3) subject to Eq. (5) should be differentiated with respect to $t_2/t_1$ at fixed $K_c$, $E'$, $\varepsilon$ and $t_1$ and the result set to zero to determine the value of $t_2/t_1$ that optimizes the threshold strength for a laminar system with uniform elastic properties. (The optimal threshold strength for a laminar ceramic system with heterogeneous elastic properties must await an exact result for that case equivalent to Eq. (3).) The result for the system with uniform elastic properties is

$$\frac{2\left(1+2\frac{t_1}{t_2}\right)^{\frac{3}{2}}\sqrt{\frac{t_1}{t_2}}}{\left(1+\frac{t_1}{t_2}\right)^2}-\frac{4}{\pi}\sqrt{\frac{1+2\frac{t_1}{t_2}}{1+\frac{t_1}{t_2}}}=\frac{K_c}{(-\varepsilon E')\sqrt{\pi t_1/2}} \qquad (7)$$

A second differentiation of Eq. (3) has been carried out to check that this result does give a maximum for the threshold strength.

As noted above, the formula in Eq. (3) for the threshold strength is only valid if Eq. (2) is satisfied. Equation (2) can be combined with Eqs (5) and (7) to show that the optimization of the threshold strength being attempted can only be carried out in the manner used to obtain Eq. (7) if

$$\frac{t_2}{t_1} \geq \pi^{2/3} - 1 = 1.145 \qquad (8)$$

which requires that

$$\frac{K_c}{(-\varepsilon E')\ \sqrt{\pi t_1/2}} \leq 0.883 \qquad (9)$$

The result in Eq. (7) has been evaluated numerically and plotted in Fig. 2 subject to the validity limitations of Eqs

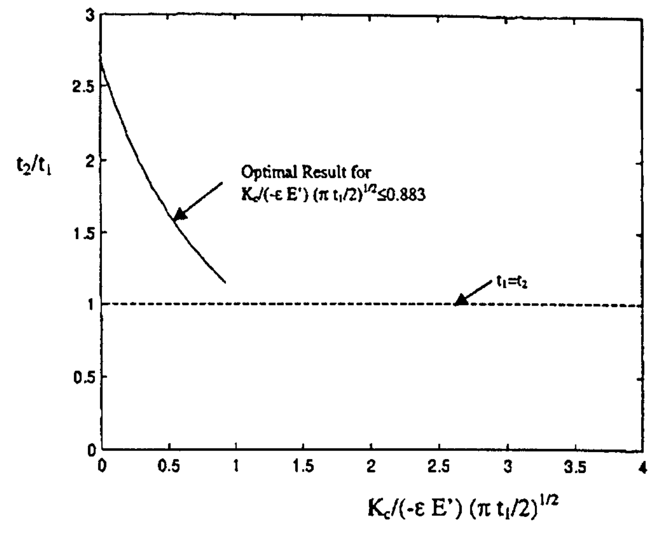


Fig. 2. The layer thickness ratio which optimizes the threshold strength when the thickness $t_1$ of the compressive layer has a fixed value. When $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is greater than unity, the layer thicknesses for optimal threshold strength are chosen to be equal and as small as is practically possible, as indicated by the dashed line.

(8) and (9). The line in Fig. 2 has been marked according to Eq. (9). It can be seen that the ratio $t_2/t_1$ is a few times larger than unity when the toughness is low, the residual stress is high or the compressive layer thickness is large. In this situation, the residual stress contribution in Eq. (3) is dominant and so optimizing that is important. This leads to the tensile layer thickness being a few times larger than the compressive layer thickness to give the best result. On the other hand, when the toughness is high, the residual stress is small or the compressive layer is thin, the toughness contribution in Eq. (3) is dominant and the highest threshold strength is achieved by making the tensile layer as thin as possible. As the toughness rises, the residual stress falls or the compressive layer thickness falls, there is a gradual transition from the case in which the residual stress dominates in Eq. (3) to the one in which the toughness term dominates. Following this trend, the layer thickness ratio $t_2/t_1$ for optimal threshold strength gradually falls from 2.5 to just above unity, at which stage the limitations of Eqs (8) and (9) are reached.

For the cases where the optimal threshold strength can be achieved with the unconstrained value of $t_2/t_1$ predicted in Eq. (7), the predicted optimal threshold strength has been calculated numerically from Eq. (3) and plotted in Fig. 3, marked according to the limitation in Eq. (9). It can be seen that when the toughness is low, the residual stress is high or the compressive layers are thick (i.e. the residual stress dominates the threshold strength predicted in Eq. (3)), the threshold strength is somewhat less than the effective residual stress, $-\varepsilon E'$. However, as the toughness rises, the residual stress falls or the compressive layers become thinner, the ratio of the threshold strength to the effective residual stress rises. This takes place as the transition occurs in Eq. (3) from the residual stress dominating the threshold stress prediction to the toughness doing so.

To obtain an optimal threshold strength for regimes violating Eq. (9), a different formulation must be developed. In this case, it must be recognized that unstable crack growth occurs before the crack has penetrated all the way through

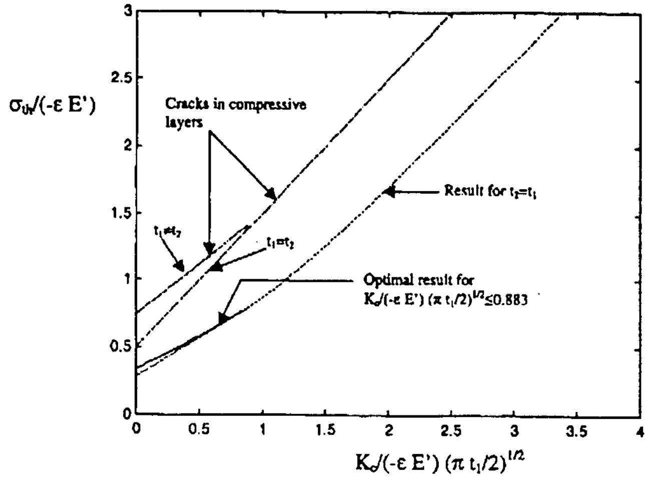


Fig. 3. The optimal threshold strength of a laminar ceramic. The thickness $t_1$ of the compressive layer should be chosen to be as small as possible and that of the tensile layer, $t_2$, selected according to Fig. 2. The threshold strength is then given by the full line when $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is smaller than 0.883 and by the dashed line when $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is greater than unity. The dashed line is also the non-optimized threshold strength for any laminar system with homogeneous elastic properties when the compressive and tensile layers have the same thickness, but that thickness is arbitrary. Also shown are threshold strengths calculated for cracks in compressive layers. These lie above the equivalent results for cracks in tensile layers, confirming that the latter control the true threshold strength in laminar ceramics.

the compressive layer. A full treatment of the resulting optimization problem is rather involved and will not be attempted. However, for completeness, the threshold strength for cases violating Eq. (2) will be developed to augment the result for situations conforming to Eq. (2) already given by Rao et al. [1].

## 4 Threshold Strength for High Toughness Laminar Ceramics

When Eq. (2) is not met by a laminar ceramic, failure of the cracks initially in the tensile layers occurs by unstable propagation before they penetrate all the way through the compressive layers. The instability is associated with a minimum in the stress intensity factor as the crack propagates through the compressive layer. After the minimum is encountered, the stress intensity factor will begin to rise as the crack grows at fixed applied stress. This will cause unstable growth of the crack and therefore failure of the laminar ceramic.

The derivative of Eq. (1) subject to Eq. (5) gives

$$\frac{dK}{da} = \frac{1}{2a}\left[K + \varepsilon E'\sqrt{\frac{a}{\pi}}\,\frac{2t_2/a}{\sqrt{1-(t_2^2/4a^2)}}\right] \tag{10}$$

Instability occurs with this equal to zero while $K = K_c$ and the crack half-length meeting this will be designated $a_c$. It follows that the crack length $2a_c$ at instability satisfies

$$\sqrt{\frac{2a_c/t_2}{(4a_c^2/t_2^2)-1}} - \frac{\pi K_c}{4(-\varepsilon E')\sqrt{\pi t_2/2}} \tag{11}$$

The threshold strength is then given by

$$\frac{\sigma_{th}}{(-\varepsilon E')} = -\frac{K_c}{(-\varepsilon E')\sqrt{\pi a_c}} + \frac{1}{1+\frac{t_1}{t_2}} - \frac{2}{\pi}\sin^{-1}\left(\frac{t_2}{2a_c}\right) \tag{12}$$

Eq. (11) can in fact be solved for $2a_c/t_2$ as a quadratic equation and the result then inserted into Eq. (12). However, the consequent expressions are tedious and not particularly illuminating. This procedure would however yield the expressions required for the unconstrained optimization of the threshold strength at fixed $t_1$ beyond the limitations imposed by Eqs (8) and (9). The procedure needed to carry out this optimization is rather messy and yields only modest amounts of information compared with the effort involved.

## 5 Threshold Strength for Laminar Ceramics with Layers of Equal Thickness

A little more insight is gained by addressing the case where $t_1 = t_2$. In this situation, Eqs (11) and (12) are valid for $1 \le 2a_c/t_2 \le 3$ and thus for $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \ge 0.780$. When the latter condition is not met, Eqs (3) and (5) are used to obtain the threshold strength when $t_1 = t_2$. Therefore, for $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \le 0.780$, the threshold strength when $t_1 = t_2$ is given by

$$\frac{\sigma_{th}}{(-\varepsilon E')} = 0.284 + \frac{K_c}{(-\varepsilon E')\sqrt{3\pi t_1/2}} \tag{13}$$

All the results for the threshold strength when $t_1 = t_2$ have been computed numerically and the result plotted in Fig. 3 as the dotted line marked "Result for $t_2 = t_1$".

## 6 Optimal Threshold Strength for Laminar Ceramics with Minimal Layer Thickness

Inspection of Eq. (3) shows that it is possible to increase the threshold strength as much as possible by making the thicknesses of the layers as small as feasible. The question of how thin the layers can be made is a technological matter concerning the processing of the laminar ceramics. Whether slip casting, solid state sintering, colloidal processing or other methods are used, the process involved will have a lower limit on the layer thickness that can be achieved. This constraint determines the possible values of $t_1$ and $t_2$. We will assume that the lower limits on both $t_1$ and $t_2$ are identical.

Consideration of Eqs (3) and (5) indicates that the term on the right-hand side of Eq. (3) containing the toughness is made as large as possible by using both $t_1$ and $t_2$ at their smallest possible values, which implies that $t_2/t_1$ is equal to 1. On the other hand, the term on the right-hand side of Eq. (3) containing the residual stress is made as large as possible when $t_2/t_1$ is equal to approximately 2.8; i.e. greater than 1. Therefore, as the toughness increases or the residual stress falls, the value of $t_2/t_1$ giving the optimal threshold strength will gradually evolve from approximately 2.8 to unity. This means that $t_2/t_1$ for optimal threshold strength is always greater than or equal to unity and values of $t_2/t_1 < 1$ need never be considered. The strategy for optimizing the threshold strength is therefore to select $t_1$ at its minimal practical value and then vary $t_2/t_1$ to maximize the threshold strength, subject to $t_2/t_1 \ge 1$. This procedure is identical to that used for unconstrained optimization at a fixed value of $t_1$, except that now the constraint $t_2/t_1 \geq 1$ must be invoked where appropriate. As a consequence, the optimal results plotted in Figs 2 and 3 with $t_2/t_1 \geq 1.145$ (i.e. for $K_c/(\varepsilon E')\sqrt{(\pi t_1/2)} \leq 0.883$) are still valid but now with the interpretation that $t_1$ is chosen to be at its smallest possible value rather than some arbitrary level.

The optimal threshold strength for the range in which $1 \leq t_2/t_1 \leq 1.145$ requires use of the results for systems in which unstable crack propagation occurs before the flaw has penetrated all the way through the compressive layer, i.e. Eqs (11) and (12). As noted above, the optimization of these results involves some messy calculus and has not been attempted. In any case, the usefulness of these results is limited since they would fill only a small gap in the results. Glossing over this omission, we turn to the case where $t_1 = t_2$ which will prevail in the optimal situation for high values of $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$, given that $t_1$ has been chosen at its smallest practical value. The choice of layer thickness ratio yielding the optimal threshold strength thus can be deduced from Fig. 2 by combining the predictions of the optimal result for $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \leq 0.883$ (plotted as the full line) with $t_2 = t_1$ for values of $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ above unity (plotted as a dashed line). In the segment where there is a gap between the lines in Fig. 2 representing these two results, an interpolation can be used to move smoothly from one to the other.

Since we have already dealt with the optimal threshold strength when $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \leq 0.883$, we are only concerned with situations other than this when the optimal strength is associated with $t_2 = t_1$. The results of the previous section show that the threshold strength in this regime is given by Eq. (12) with $t_2 = t_1$, subject to Eq. (11). Differentiation of Eq. (12) with respect to $t_2$, while $a_c$ is defined by Eq. (11) as a function of $t_2$, shows that the threshold strength is made optimal in this situation by choosing the layer thicknesses to be as small as practically possible. (This step is laborious and the result is not surprising, so we do not present the details. However, it shows that *ceteris paribus*, the threshold strength increases monotonically to infinity as $t_2$ approaches zero. Simultaneously, $2a_c$ approaches $t_2$ from above, showing that when the layers are very thin, instability of the crack growth occurs when the flaw has barely penetrated the compressive layer.) The important result from this consideration is that when $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is somewhat larger than unity, the optimal threshold strength is obtained by making the compressive and the tensile layer thicknesses equal and as small as is practically possible. The optimal threshold strength for $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ somewhat greater than unity is then given by the result for $t_2 = t_1$ plotted in Fig. 3 as a dashed line, but with the implication that $t_1$ has been chosen to be its smallest practical value.

The optimal threshold strength for any value of $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ when $t_1$ has been chosen as its smallest practical value thus can be obtained by using the optimal result for the range $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \leq 0.883$ (plotted as the full line in Fig. 3) combined with the result for $t_2/t_1$ (plotted as a dashed line in Fig. 3) used when $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is greater than unity. The gap between $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ equals to 0.883 $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ somewhat greater than unity can be handled by interpolation. Figure 3 makes clear that little interpolation is needed since the two results are almost equal at $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} = 0.883$.

We have also plotted in Fig. 3 the threshold implied by cracks initially in the compressive layers. These results are calculated from Eq. (4) with Eq. (5) used to calculate the compressive residual stress. The relevant lines are marked "Cracks in compressive layers." One result is obtained by using the optimized layer ratio from Fig. 2 in the range $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)} \leq 0.883$ and is marked "$t_1 \neq t_2$" in Fig. 3. It can be seen that this prediction lies above the optimal result for cracks in tensile layers, confirming that the latter is the true threshold as mentioned previously. Another result is obtained for cracks in compressive layers of laminar ceramics with equal layer thicknesses and is marked "$t_1 = t_2$" in Fig. 3. This latter case is obtained for the entire range of Fig. 3 and it can be seen that it lies above the result for cracks in tensile layers of materials with equal layer thicknesses. This observation confirms the latter result as the true threshold in all laminar ceramics with layers of equal thickness.

## 7 Discussion

As an example, we consider the case of partially stabilized zirconia sandwiched between layers of unstabilized zirconia which have been bonded together at high temperature and then cooled down below the transformation from tetragonal to monoclinic for the unstabilized material [10]. The stabilized zirconia will not transform, so a volume difference of approximately 3 % will exist between the two different layers [10]. This implies that $\varepsilon$ equals $-1$ %. All relevant mechanical properties of the system are homogeneous, which means that our optimal results are valid. Young's modulus of zirconia is 200 GPa and Poisson's ratio is approximately 0.2. Consequently, $E'$ is approximately 250 GPa. The toughness of the compressive layer (the unstabilized zirconia) is taken to be 3 MPa$\sqrt{\text{m}}$. Say, the layers can be made down to a thickness of 5 µm, which is perhaps ambitious. The parameter $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is then equal to 0.43. Figure 2 shows that for optimal threshold strength, the tensile layers of stabilized zirconia should be made approximately 70 % thicker than the compressive layers of unstabilized zirconia. Figure 3 indicates that the optimal threshold strength is approximately half of the effective residual stress, which is 2.5 GPa. Therefore, the optimal threshold strength is a very respectable 1.25 GPa. If the layers can only be made as thin as 50 µm, $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is approximately 0.14. From Fig. 2, a tensile layer thickness of about 2.2 times the compressive layer thickness is indicated. Figure 3 shows that the optimal threshold strength is about 40 % of the effective residual stress, or 1 GPa. Say the zirconia is present in a composite at 15 % by volume in another ceramic having the same Young's modulus. The value of $\varepsilon$ is then approximately $-0.15$ %. With a compressive layer thickness of 5 µm, $K_c/(-\varepsilon E')\sqrt{(\pi t_1/2)}$ is equal to 2.9. As indicated in Fig. 2, equal layer thicknesses are called for to obtain the optimal threshold strength. Figure 3 shows that the optimal threshold strength is approximately 2.5 times the effective residual stress, which is now 0.375 GPa. As a result, the optimal threshold strength is 0.94 GPa.

It should be noted that the threshold strengths discussed here depend on the mechanism of failure. It has been observed [10] that the cracks in the tensile layers sometimes bifurcate and run parallel to the interfaces rather than growing into the neighboring compressive layer. Presumably this occurs at an applied stress below the threshold predicted for cracks which do penetrate the compressive layer, but the threshold strength measured in the bifurcating cases is little different from that observed otherwise. A satisfactory model has not yet been developed for the bifurcating cracks, so an optimal threshold strength for this situation cannot yet be predicted. However, it seems clear that a threshold strength of significant magnitude does exist in this case. Apart from this qualification, the model of optimal threshold strength presented in this paper indicates the scale of strength possible from the approach of layering compressive and tensile slabs together. From the example just presented, it can be seen that the strengths can be quite significant.

## 8 The New Millenium

As requested by the editor of this special edition of *Zeitschrift für Metallkunde,* Professor Dr. Manfred Rühle, we present a few thoughts on the topic of our paper concerning its development in the next millenium. In our case, this is a particularly easy task, since it seems to us that Fred Lange's discovery of the phenomenon of a threshold strength in laminar ceramics will revolutionize the use of ceramics in structural applications and other situations where a reliable strength is required. The task for researchers is to develop this opportunity and identify and obviate the pitfalls, problems and limitations which will undoubtedly present difficulties in the implemention of this approach for developing strong ceramics. For example, damage induced during processing in systems with very thin layers will probably cause cracks that span several slabs of the material. This does not obviate the advantages of the laminar ceramic approach, but rather limits the effectiveness of using very thin layers. However, this scenario can be explored experimentally and theoretically and the extent to which it underlines the value of Fred's discovery investigated. In addition, ceramicists adept at processing will undoubtedly be able to improve the quality of the laminar systems which can be produced, including the tendency for large processing flaws to be introduced.

A more positive note is that Fred's pioneering work will lead undoubtedly to follow-up developments. Researchers will exploit his basic idea to endow ceramics with robust threshold strengths against failure mechanisms other than those involved in laminar ceramics loaded parallel to the interfaces.

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*(Received September 3rd, 1999)*



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# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0705–0188

| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | Feb 10, 2000 | Technical Repts. # 2<br>5/1/99 through 4/30/00 |

| | |
|---|---|
| 4. TITLE AND SUBTITLE<br>Reliable Ceramic Structural Composites Designed with a Threshold Strength | 5. FUNDING NUMBERS<br>N00014-99-1-0638 |
| 6. AUTHOR(S)<br>PII: Fred F. Lange | |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>Materials Department<br>College of Engineering<br>University of California<br>Santa Barbara, CA 93106-5050 | 8. PERFORMING ORGANIZATION REPORT NUMBER<br>NA |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)<br>Office of Naval Research<br>Ballston Centre Tower One<br>800 North Quincy Street<br>Arlington, VA 22217-5660 | 10. SPONSORING/MONITORING AGENCY REPORT NUMBER |

11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an offician Department of the Army position, policy or decision, unless so designeated by other documentation.

| 12A. DISTRIBUTION/AVAILABILITY STATEMENT | 12B. DISTRIBUTION CODE |
|---|---|
| Approved for public release; distribution unlimited. | |


13. ABSTRACT (Maximum 200 words)

Laminar ceramics in which alternating layers of materials are bonded together exhibit a threshold strength if one set of layers has a compressive, residual stress. The threshold strength is substantial if the mechanical properties of the component ceramics are chosen wisely. Optimization of the threshold strength of a system with homogenous elastic properties in terms of layer thickness, fracture toughness and residual stress is presented. The best result is shown to be associated with the toughest material and the highest residual stress, but the choice of the laminar system exhibiting such features is limited to available ceramics. For each material system, the threshold strength is further optimized by making the layers as thin as possible. The thinness achievable will be limited by the technological processes used to make the laminar ceramic and material stability. If the system mechanical properties are favorable, the optimized threshold strength can be comparable in magnitude or much larger than the compressive, residual stress.


| 14. SUBJECT TERMS | 15. NUMBER OF PAGES<br>7 pages |
|---|---|
| | 16. PRICE CODE |

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | UL |